Evidence for V363 Cas as a First Overtone Anomalous Cepheid

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V363 Cassiopeiae was observed through 51 acquisitions of each of V, B, i and z filters, during a 15 day observation window. From the observations, folded light curves were generated using a PDM technique. It was my objective to provide further evidence for this star's reclassification as a first overtone Anomalous Cepheid, as some past papers have proposed (Fernley, 1998). Based on our light curve characteristics (shape, and period), V363 Cas appeared to favor the anomalous Cepheid class over any RR Lyrae class. My observed period of 0.545 days is higher than the typical range of periods for RRd Lyrae, reported between 0.25 and 0.49 days (Soszynski et al., 2008). The RRab type Lyrae, as some have imposed on V363 Cas (Kholopov et al., 1985), was ruled out due to the evidence for overtone pulsation by Hajdu et al. (Hajdu, Jurcsik, et al., 2009) and Fernley (Fernley, 1998). Finally, a rough distance comparison to GAIA, using Nemec's 1994 P-L-[Fe/H] for Anomalous Cepheids (Nemec, Nemec, & Lutz, 1994), estimated the distance of V363 Cas to be closer to the distance estimated by GAIA than estimates made with RRd class equations.

Keywords: *Stars: variables: RR Lyrae – stars: variables: Cepheids – stars: distances*

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INTRODUCTION

The classification of V363 Cas has been questioned in almost every successive study since its original designation as an RRab Lyrae star (Kholopov et al., 1985). In 1996, it was proposed that it could be an RRc instead, based on the light curve shape (Schmidt & Seth, 1996). In 1998, Fernley proposed it could be a first overtone Anomalous Cepheid after finding harmonic evidence in its period through Fourier analysis (Fernley, 1998). Then, in 2009, Hajdu et al. generated much clearer data on its light curve and deemed it to be closer to a short period Cepheid class (Hajdu et al., 2009).

Perhaps one of the most compelling reasons to accurately classify V363 Cas is to utilize its pulsation period and luminosity (and sometimes metallicity) relationship in order to derive a distance estimate. Catelan and Smith advocate for the usefulness of RR Lyrae stars as standard candles in our understanding of distances to distant star systems, such as globular clusters (Catelan, Pritzl, & Smith, 2004). Outside of RR Lyrae, other astronomers, like Nemec (Nemec et al., 1994), also advocate for Anomalous Cepheids (AC) as useful distance indicators for the same reasons, in addition to the fact that AC's are more luminous than RR Lyrae. Undoubtedly, the class of the star is important to estimating an accurate distance calculation.

The aim of this paper is to attempt to bring added resolve to the classification of V363 Cas, and perhaps to provide information on a particularly spurious variable that appears to tread the boundaries between RRc type Lyrae stars and anomalous Cepheid stars. There still appears to be disagreement on those boundaries; Groenwegan and Jurkovic (2017) state ACs tend to have periods from 0.9 - 2 days, while Nemec (1988) claims they can have periods as short as 0.26 days and as long as 2.37 days. That said, Jurkovic (2019) greatly widened the estimated pulsation period range of ACs to 0.24 - 4 days shortly after the earlier publication with Groenwengen (2017). With this paper, the hope is that more could be learned about the limits of each class to aid future classifications of variable stars.

OBSERVATIONS

Acquisitions in B, V, i', and z' bands were taken of V363 Cas over 15 days, in between September 29 and October 13 2019. These were taken using the SBIG 0.4m telescopes available at the Las Cumbres Observatory network (Brown et al., 2013). These telescopes feature a 0.571" pixel in a 1x1 bin mode, and a 29.2 x 19.5 arcmin field of view. One observation from each filter was taken upon each acquisition through the 15 day observation window. A total of 51 images were successfully taken from 65 attempts. 14 of the acquisitions failed due to visibility issues such as cloud cover. Exposure times were empirically chosen, based on test images taken prior to the observations presented within this research paper. For each filter, 1 test image was taken, each with a 30s exposure. Using the Source-Sky tool in AstroImageJ, the pixel values were collected. From there, exposures were re-calculated to attempt to achieve 100,000 counts. This is expressed in table 1.

Filter	Source-sky, test images	Exposure	Optimized exposure for 100,000 counts
B	75,602	30s	39.68 = ∼40s
V	254,766	30s	11.77s =∼12s
i'	290,673	30s	10.32s =∼10s
z'	78,515	60s	76.45c = ∼76s

Table 1. Optimized exposure values used in this research, for each respective filter

METHODS

Brightness Calibration of Light Curve

From the successful images obtained for each filter (B, V, i, z), the relative pixel counts of V363 Cas were calibrated to brightness values by using reference stars in the field of view around V363 Cas, of which had readily available apparent magnitudes. The calibration procedure is as follows.

The images were first automatically processed into photometry files using the Solar Siblings Pipeline (Fitzgerald, 2018) to generate PSFEx photometry files (Bertin & Moneti, 2017). Point spread function (PSF) photometry was chosen because the field of view contained a great number of stars, some of which appeared to overlap in some of the clearest images. Although, after completing the PSF method, aperture photometry was also done in the V band; it gave strong agreement with the PSF method, indicating that aperture photometry would have been an equally suitable method for this study.

The RA and DEC were estimated inside of Aladin Sky Atlas (Bonnarel et al., 2000), using one of the 51 images. The coordinates were found to be 3.8095, +60.3404 (RA, Dec, respectively, in degrees). Once more precise coordinates were known, neighbouring stars which were between 2000 and 1000000 pixel counts were selected as possible reference stars. Of those, the known variable stars were found, and removed from the reference star list. The apparent magnitudes for the remaining non-variable reference candidates were found in available APASS (Henden et al., 2016), and SDSS (Ahn et al., 2012) catalogue data. Those magnitudes are shown in the appendix tables, based on coordinates. These were used to calibrate the apparent magnitude of each image of V363 Cas. Each of the successful images used were then calibrated into magnitude, using the standard calibration stars discussed above. This was repeated for each of the filters (B, V, i, and z). The error associated with this method stems mainly from the previously reported errors in the data catalogues, as well as with the random sampling error from each of the 51 images.

Period Estimation of Light Curve

With the calibrated images, folded light curves were generated for V363 Cas. The phase dispersion minimization (PDM) method (Paunzen & Vanmunster, 2016) was used to generate the light curves; this is a statistical method in which test period values are compared to find the one which generates the least amount of scatter. This was chosen because it is a long accepted method suitable for situations with limited observations, and for non-sinusoidal light curves, much like the asymmetrical curves of RR Lyraes and Cepheids (Stellingwerf, 1978). Stellingwerf demonstrates the effectiveness of this method in a complex case of the double-mode Cepheid BK Cen, even with only 49 data points. Due to the limited images and observation time from our study, this appeared to be a suitable method.

To apply the PDM method, a python package called astrosource (Fitzgerald, Gomez, Salimpour, Singleton, & Wibowo, 2021) was utilized; Astrosource imports the PSFEx files and identifies the least variable reference stars around the target star across the full data set with respect to each filter (in this case B, V, i', and z'). These reference stars are all presented by coordinates in the appendix tables. It uses these reference stars to calibrate the average brightness of our target star. In Astrosource, 10 000 trial periods were assessed in between between 0.2 and 1.2 days, with a bin width of 10. The output of this is a probability graph plotted over the possible periods, whereby the probability is indicated by the variance of all 10 000 trials. In figure 1, the probability graph is given for the visible filter as an output from Astrosource. The error associated with the period was estimated by assessing either side of the center of the peak, out to where either side of the peak dropped to 0.95 of the peak period value. This error could be reduced with more observation images, making the folded light curve more certain. The light curves are shown in the Results section.

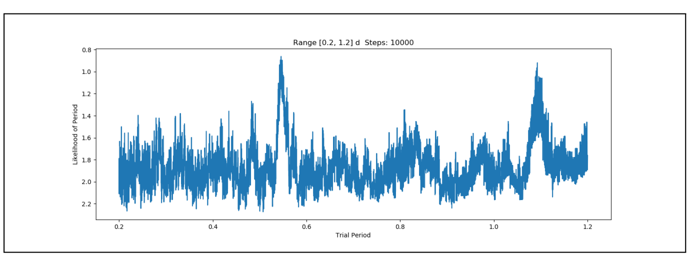

Fig. 1. Probability plot of the period, using the PDM method on V acquisitions

RESULTS

Light Curves

The light curve generated from the methods discussed in sections 3.1 and 3.2 are shown in figures 2, 3, 4, and 5.

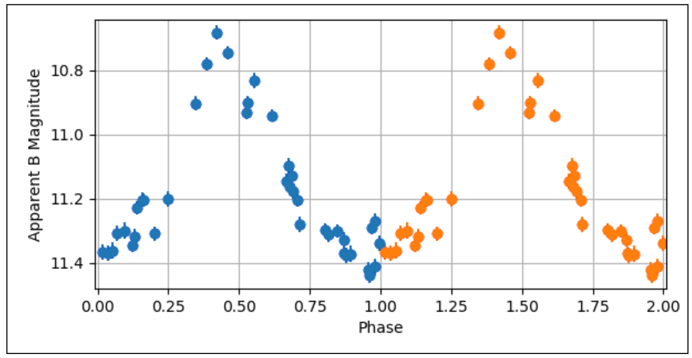

Fig. 2. Folded light curve from the B filter showing an average magnitude of 11.061 +/- 0.1.

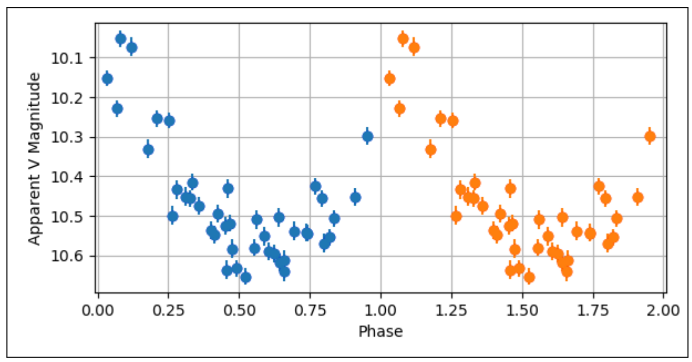

Fig. 3. Folded light curve from the V filter showing an average magnitude of 10.352 +/- 0.08.

A summary table of the periods derived from each filter set are shown in table 2.

Brightness

The brightness derived from the calibration in section 3.1 is shown in table 3. The value of apparent magnitude is calculated as the midpoint brightness value halfway between the minimum and maximum brightness, of the 51 processed images. The amplitude is the difference between the minimum and maximum values.

DISCUSSION

Comparison To Existing Observations

The processed data in table 3 was then compared to previous observations of brightness. APASS figures

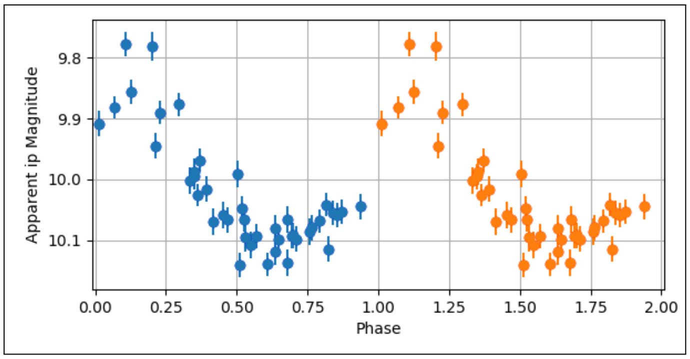

Fig. 4. Folded light curve from the i filter showing an average magnitude of is 9.959 +/-0.07.

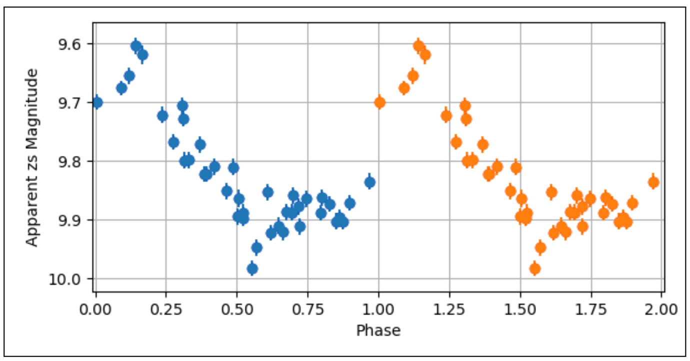

Fig. 5. Folded light curve from z filter showing an average magnitude of 9.793 +/-0.1.

Filter	Period, PDM	PDM Error
B	0.5475	0.00915
V	0.5458	0.0076
I	0.5463	0.0089
Z	0.5444	0.00995
[Average], [error]	0.546	0.0045

Table 2. The period values above all all expressed in therms of days. From the data, the average period across the four filters is 0.546 days, with an uncertainty of 0.0045 days.

Filter	Brightness (magnitude)	Amplitude (max - min, in mag)	Error (Mag)
B	11.061	0.756	0.1
V	10.352	0.601	0.08
i	9.959	0.364	0.07
z	9.793	0.378	0.1

Table 3. Calibrated brightness values and error according to their associated filter band. Also included is the light curve amplitude.

report B and V brightness magnitudes of 11.192+/-0.224, and 10.488 +/-0.169, respectively. This is in agreement of our numbers. Finally, Seth and Smith (1996) report a V magnitude of approximately 10.5. Hajdu et al. (2009) report a V brightness of 10.569. Those aforementioned values are higher than our observations, although error ranges are not provided. HOG E et al. (2000) reports a B brightness of 11.26+/-0.06; however, their Tycho-B filter differs from the Johnson-B filter used in this study and is therefore not a comparable value. The magnitudes in i and z were not found in the existing literature for V363 Cas.

From the calculations shown in table 2, Hajdu et al. (2009) report a period of 0.54655 +/-0.011 days, a strong agreement with our calculated period.

Classification of V363 Cas

Since the discovery of variability in V363 Cas in 1959 (Nowakowski, 1988), the classification has been continually challenged with no clear agreement on the class. Nowakowski (1988) cited it as an RR Lyrae. Then, Schmidt and Seth (1996) classified it specifically as an RRc Lyrae . Fernley et. al. (1998) suggest it has evidence of being a first-overtone anomalous Cepheid. Finally, Hajdu et. al (2009) consider it to be a short period Cepheid.

From this study, the metallicity, light curves, and brightness values all individually cast some doubt on it residing in the RR Lyrae class; however, the most compelling evidence is given by the distance calculations using Nemec's period-luminosity-metallicity (P-L-[Fe/H]) equation (1994) compared with other fits using RR Lyrae P-L equations from Catelan (2004). Before showing these comparisons, some critical spectral data, light curves, and period are first presented, as this data is critical to the distance calculations.

The only spectral data on V363 Cas are delta S values from Willis (1972), providing Sp(H) and Sp(K) values of F7, and F6 respectively. Using Layden's Eq. (6) delta S-[Fe/H] calibration (1994) for RR Lyrae stars, the metallicity was derived as follows:

$$[Fe/H]_{ZW} = (-0.144 * dS) - 0.54 \quad (1)$$

$$[Fe/H]_{ZW} = (-0.144 * 1.0) - 0.54 = -0.68 \quad (2)$$

From Layden's study of 302 RRab in 1994 (1994), the mean metallicity was found to be -1.42 dex, with a standard deviation of 0.53 dex. This places V363 Cas 1.4 standard deviations away from the mean type RRab Lyrae from this study, based on Layden's assessment. The metallicity reported by Willis is therefore on the fringe of what is expected from an RR Lyrae, and even that of Anomalous Cepheids. RRc Lyrae observed by Walker and Terndrup (1991) appear to peak at -1 dex, with a standard deviation of 0.16 dex; compared to their expected value, V363 Cas would be 2 standard deviations from the normal distribution. According to Jurkovic (2019), a low metallicity is fundamental to the evolution of anomalous Cepheids, which is supported by the 4 samples discussed in that study (albeit a low statistical number).

With the relatively more metal-rich result of Willis' data, metallicity alone is not a satisfactory determinant of V363 Cas' class. For added closure, a promising avenue in future research on V363 Cas would be to re-evaluate the metallicity of V363 Cas with modern equipment. Based on the light curve shapes illustrated in section 4.1, we can see that the curves show a nearly symmetrical shape and have a sharp maximum occurring slightly before halfway through the phase. This is most clearly shown in figure 2 of the light curve acquired in the blue filter. Before comparing to other light curve shapes, we must acknowledge that previous fourier analysis (Fernley, 1998)(Hajdu et al., 2009) provide evidence that V363 Cas pulsates in its first-overtone mode. Thus, it is not worth comparing it to its class as an RRab type Lyrae, as those are fundamental pulsators. RRc type Lyrae, on the other hand, are 1st overtone pulsators, and their light curves show a nearly symmetrical shape, primary with rounded maxima, and sometimes sharper maxima (Soszynski et al., 2008). When comparing light curves to first-overtone anomalous Cepheids (1OAC), such as V742 Cyg (Jurkovic, 2019), there is a high degree of similarity in the shape; the sharp maximum occurs at a phase of 0.4, much like the one shown in figure 2. In reviewing light curves from other studies, such as Hajdu et al. (2009), the light curve appears very similar to the one observed in this study, but the sharp apex and near-symmetrical shape is even more apparent with increased sampling. From this analysis, the light curves collected appear more consistently with the light curve of an anomalous Cepheid, rather than an RRc Lyrae. From a study of 4958 RRc Lyrae stars in the LMC, Soszynski et al. (2008) calculated RRc type Lyrae to have a mean period of 0.337 days, and a total range of 0.25 to 0.49 days. This would statistically place V363 Cas outside of the RRc range. Furthermore, the observed period of V363 Cas would be agreeable to the ranges expressed by Nemec (1988) and Jurkovic (2019). Thus, the period gives some further justification of V363 Cas' classification as a 1st overtone Anomalous Cepheid.

Absolute Magnitude Calculations

Using Period-Luminosity-Metallicity relationships (P-L-[Fe/H]) established by previous papers (Nemec 1994, Pritzl et al. 2002, Catelan 2004, Catelan 2008), I generated a magnitude for V363 Cas. The challenge was using the correct classification, as P-L-[Fe/H] each depend highly on the class of star. The magnitude in the V filter was compared using a few different equations across the potential classes, shown in table 4.

From the values in table 4, it can be see that the RR Lyrae classification imposes an absolute magnitude of approximately 0.85 to 0.9. From a 1996 study on RR Lyrae stars, those with a metallicity of -0.76, similar to our chosen metallicity value, would be expected to have an absolute magnitude of 0.79 +/- 0.3 mag (Layden, Hanson, Hawley, Klemola, & Hanley, 1996). The calculated magnitude values in table 4, using the RR Lyrae class formulae, are well within the statistical range of Layden's values. Using Nemec's Harmonic Anomalous Cepheid (AC,H) formula for M_v, the magnitude is lower (Nemec, 1994). To try and distinguish the likelihood of the class, the above values were compared to the GAIA distance measurement in section 5.4.

Distance Calculations

Interstellar extinction, expressed as E(B-V), was determined using the observed magnitudes from table 3, for M_v, and the calculated values from table 4, for M_v. The values for M_b were generated the same way, us-

P-L-[Fe/H] Method	Test Class	Calculated M_v (mag)
Catelan, 2004 eq.(8)	RR Lyrae (ab)	0.856
Nemec, 1994 eq.(RRc)	RR Lyrae (c)	0.879 +/- 0.033
Nemec, 1994 eq.(AC,H)	AC, 1O	-0.035 +/- 0.084

Table 4. The test class in the above table represents the tested class based on the equation chosen in the adjacent column. No error is given in Catelan's method (2004), because it is mentioned that the error is very small and thus not presented. All methods assume the set metallicity calculated in section 5.2, and the period calculated from our observations in table 2. The range is due to the uncertainty in the period in the visible filter, expressed in table 2, as well as the uncertainty in the respective formula used.

ing the formulas provided by Nemec (1994) in terms of both RRd Lyrae, and 1st overtone AC's. An E(B-V) for both the RRd Lyrae and the AC class star classes was generated to support the distance calculations. From the magnitudes found, I obtain either an E(B-V) of 0.36 +/- 0.028 for RRd Lyrae, or 0.532 +/- 0.034 for the 1st overtone anomalous Cepheid. A second source for E(B-V) was found through the NASA/IPAC infrared science archive for our object, expressed as a maximum value of 0.9042 +/- 0.0718 (Schlafly & Finkbeiner, 2011). However, the actual reddening value at the distance of V363 Cas is expected to be between 0 and this value.

Using equations (3) and (4), and the nominal M_v values, we obtain a distance of 559±68pc for the 1OAC class, and calculation of 469±35pc for the RRd type Lyrae class. The error was generated through an RSS calculation using independent error variables from interstellar extinction in the preceding paragraphs, the period error in table 2, and the M_v error in table 4. The dominating error arises from the E(B-V) values in each case. Although neither calculation statistically agrees with the GAIA value, the calculation for V363 Cas as a 1OAC star comes closer to the GAIA value than if it were categorized as an RRd Lyrae. More confidence in an interstellar extinction value of V363 Cas would prove to be very valuable in helping to reduce this uncertainty.

$$m - M = 5(logd) - 5 \quad (3)$$

$$d = 10^{((m-M+5)/5)} \quad (4)$$

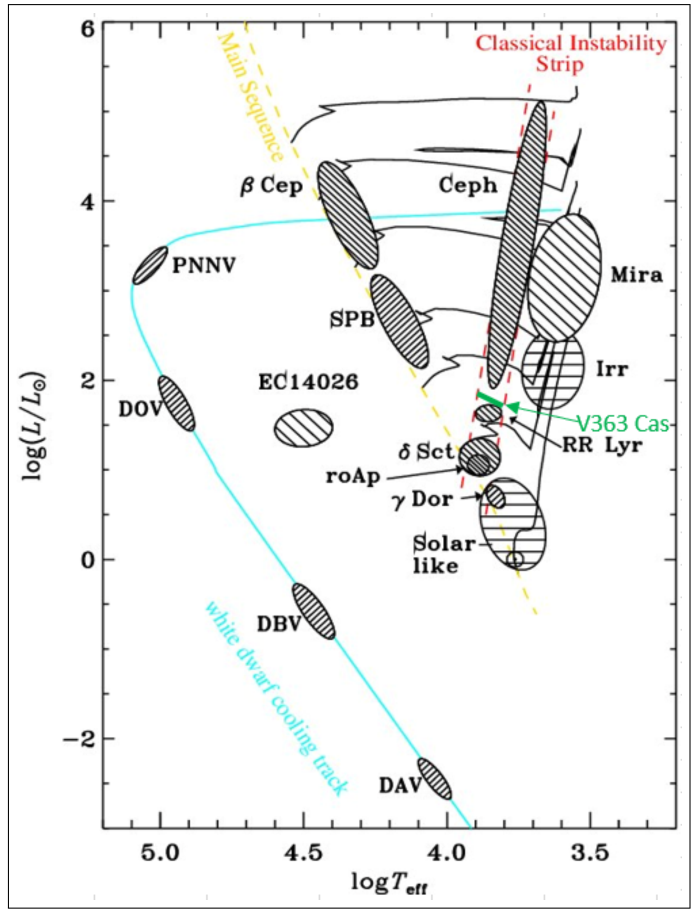

Fig. 6. Estimated location of V363 Cas on the HR diagram.

Estimated Position of V363 on the HR Diagram

From the information gathered thus far, I attempted to place V363 Cas on the HR diagram. But, for practical purposes, the GAIA value of 1257pc (Bailer-Jones, Rybizki, Fouesneau, Mantelet, & Andrae, 2018) was used to generate expected M_b and M_v values for each possible interstellar reddening extreme (0 to 0.528), for which could be converted into L/Lo. This is depicted as the line shown in figure 6.

CONCLUSION

The observations within this study largely agree with previous observations and derivations of the period, and mean apparent magnitudes. With the goal of clarifying the star's class, V363 Cas appears to favor the classification of first overtone anomalous Cepheid over an RRab or RRd Lyrae star. This conclusion is bolstered by existing evidence for its first overtone pulsation, its substantially long period (for an RRd Lyrae), its asymmetrical light curve shape, distance comparisons, and the position on the HR diagram. Further observation is still be required to more confidently close this case. It is also possible that V363 Cas resides on the fringe of both the RRd Lyrae classification and the anomalous Cepheid classification, making it such a challenge to categorize. Further study would benefit from a more accurate interstellar extinction value, not dependent on the P-L-[Fe/H] calculated magnitude (M_v, M_b) values, such as the one proposed by Uddin (2011).

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RA (deg)	DEC (deg)	V mag	Err (mag)
3.53719	60.2228815	10.827	0.092
3.570382	60.2006619	12.764	0.094
3.583492	60.5100699	12.114	0.089
3.605644	60.4597089	11.81	0.085
3.607751	60.4748095	12.859	0.04
3.663101	60.5133308	12.413	0.084
4.078323	60.4066467	11.78	0.084
4.046512	60.3560583	12.581	0.089
4.040689	60.4030093	11.836	0.087
3.750878	60.4372581	10.96	0
3.730725	60.5232573	11.357	0.089
4.012417	60.3014144	10.425	0.088
3.994707	60.5036708	12.455	0.085
3.778473	60.5512899	12.494	0.089
3.890613	60.5249879	12.311	0.085
3.870875	60.1360016	11.981	0.084
3.864696	60.5086149	11.829	0.092
3.795354	60.5386262	12.435	0.088

Table 5. Stars with known V-filter brightness used to calibrate our own star's V-filter brightness

REFERENCE STARS

RA (deg)	DEC (deg)	B mag	Err (mag)
3.5372	60.2229	11.86499977	0.118
3.5834	60.5101	12.67700005	0.122
3.5742	60.1274	11.65499973	0.115
3.6056	60.4597	12.06099987	0.112
3.6212	60.5587	12.57800007	0.123
3.6631	60.5134	12.82999992	0.11
3.6732	60.4943	13.04800034	0.102
4.0784	60.4066	12.01900005	0.114
4.0465	60.3561	13.08699989	0.111
4.0407	60.4030	12.42500019	0.11
3.7785	60.5513	12.88099957	0.119
3.7508	60.4373	10.98700047	0
3.7307	60.5233	12.42399979	0.138
4.0125	60.3014	11.38099957	0.116
3.9948	60.5037	12.75399971	0.121
3.7953	60.5387	13.0369997	0.123
3.8710	60.1360	12.57499981	0.121
3.8906	60.5250	12.94400024	0.113
3.8668	60.3438	12.40499973	0.111
3.8647	60.5086	12.89700031	0.12899999
3.8231	60.1413	11.11699963	0.14300001
3.8236	60.3670	11.88799953	0.112

Table 6. Stars with known B-filter brightness used to calibrate our own star's B-filter brightness

RA (deg)	DEC (deg)	i mag	Err (mag)
3.5704	60.2007	11.762	N/A
3.5916	60.3418	7.9639	0.0031
3.5834	60.5100	11.729	N/A
3.5922	60.4745	10.1168	0.006
3.6056	60.4597	11.802	N/A
3.6630	60.5133	12.17	N/A
3.6775	60.1980	11.534	N/A
3.7023	60.2196	12.206	N/A
3.6938	60.4791	12.328	N/A
4.0783	60.4066	11.762	N/A
4.0407	60.4030	11.512	N/A
3.7508	60.4373	10.9779	0.0361
3.7307	60.5232	10.623	N/A
4.0357	60.3747	9.939	N/A
4.0124	60.3014	9.784	N/A
3.9495	60.3064	11.821	N/A
3.7784	60.5513	12.371	N/A
3.9475	60.1664	12.156	N/A
3.8905	60.5250	12.128	N/A
3.8646	60.5086	10.953	N/A
3.7953	60.5386	12.026	N/A
3.8251	60.2864	10.737	N/A

Table 7. Stars with known i-filter brightness used to calibrate our own star's i-filter brightness. Many of the errors were not found where the error is reported as "N/A"